

**Web-Based Supplementary Materials for Inference for Bivariate
Grouped Survival Data with an Application to Arm-In-Cage
Experiments by B.A. Griffin and S.W. Lagakos**

Table 1: Simulation Study Results. Estimated power of test statistics for $H_0 : F_0(t) = F_1(t)$ and $H_0 : G_1(z, t) = G_0(z, t)$ and a simple test of $H_0 : W_0(x) = W_1(x)$ which excludes information from the landings. The simulations assume hourly cage visits, no right-censoring, a Type-I error rate of 0.05, and that $T_0 \sim TN(4.5, 0.5^2, [3, 6])$ and $T_1 \sim TN(4, 0.5^2, [2.5, 5.5])$ where $TN(\mu, \sigma^2, [a, b])$ represents a truncated normal distribution with mean μ , variance σ^2 , and support $[a, b]$. Power is computed with $n_0 = n_1 = 30$ and using 300 simulations. Weibull($\lambda, \alpha_1, \alpha_2$) denotes a Weibull distribution with shape parameter λ and scale parameter $\exp\{\alpha_1 + \alpha_2 t\}$

	Null Hypothesis Tested		
	$F_0(t) = F_1(t)$	$G_1(z, t) = G_0(z, t)$	$W_0(x) = W_1(x)$
<hr/>			
$(Z_0 T_0 = t) \equiv (Z_1 T_1 = t)$			
Weibull(2, 3, -0.5)	0.94	0.73	0.07
Weibull(1, 3, -0.5)	0.90	.71	0.05
Weibull(0.5, 3, -0.5)	0.91	0.74	0.16
Weibull(2, 3, 0)	0.90	0.73	0.08
Weibull(1, 3, 0)	0.90	0.71	0.05
Weibull(0.5, 3, 0)	0.92	0.71	0.34
Weibull(2, 3, 0.5)	0.95	0.71	0.36
Weibull(1, 3, 0.5)	0.95	.72	0.13
Weibull(0.5, -5, 0.5)	0.92	0.66	0.87
<hr/>			
$(Z_0 T_0 = t) \neq (Z_1 T_1 = t)$			
$Z_0 T_0 = t \sim \text{Weibull}(2, 3, -0.5)$			
$Z_1 T_1 = t \sim \text{Weibull}(1.75, 3, -0.5)$	0.91	0.75	0.06
$Z_1 T_1 = t \sim \text{Weibull}(1, 3, -0.5)$	0.92	0.94	0.05
$Z_1 T_1 = t \sim \text{Weibull}(0.5, 3, -0.5)$	0.90	0.96	0.07
$Z_1 T_1 = t \sim \text{Weibull}(2, 2.5, -0.5)$	0.91	0.94	1.00
$Z_1 T_1 = t \sim \text{Weibull}(2, 2.5, -0.5)$	0.90	0.97	0.97
$Z_1 T_1 = t \sim \text{Weibull}(2, 2.8, -0.5)$	0.90	0.78	0.34
$Z_1 T_1 = t \sim \text{Weibull}(2, 3, -0.4)$	0.95	0.96	0.81
$Z_1 T_1 = t \sim \text{Weibull}(2, 3, -0.45)$	0.90	0.82	0.32
<hr/>			

Table 2: Power and asymptotic relative efficiency for arm-in-cage designs with $k = 6$ when $T_0 \sim TN(4.5, 0.5^2, [3, 6])$ and $T_1 \sim TN(3.5, 0.5^2, [2, 5])$. Power computed with $n = 5.39$ for each group.

Type of Design	Visit Times	Power	ARE
Class I	$[3.25, 4.5], \Delta = .25$.950	-
	$[3.5, 4.75], \Delta = .25$.950	1.00
	$[3, 4.25], \Delta = .25$.945	.973
	$[3, 5], \Delta = .25$.943	.966
Class II	$[2.5, 5], \Delta = .5$.936	.933
	$(2.25, 3, 3.25, 3.75, 4.25, 4.75)$.936	.930
	$[2, 4.5], \Delta = .5$.936	.930
Class III	$[3, 5.5], \Delta = .5$.936	.933
	$[3.5, 6], \Delta = .5$.935	.930
	$(3.25, 4, 4.25, 4.75, 5.25, 5.75)$.931	.913
Class IV	$[1, 6], \Delta = 1$.893	.788
	$[0.5, 5.5], \Delta = 1$.828	.650
Class V	$[4.75, 6], \Delta = .25$.326	.175
	$[2, 3.25], \Delta = .25$.326	.175
	$[5.2, 6], \Delta = .2$.105	.036
	$(0.5, 1.5, 2, 2.5, 5.5, 6)$.077	.018
	$[0.25, 1.50], \Delta = .25$.050	.000

*When visit times are evenly spaced, they are described by the notation $[a, b], \Delta$ where a and b denote the first and last cage visit and Δ denotes the length of time between cage visits.

Table 3: Power and asymptotic relative efficiency for arm-in-cage designs with $k = 6$ when $T_0 \sim TN(4.5, 0.5^2, [3, 6])$ and $T_1 \sim TN(2.5, 0.5^2, [1, 4])$. Power computed with $n = 0.456$ for each group.

Type of Design	Visit Times	Power	ARE
Class I	$[3, 4], \Delta = .2$.950	-
	$[3.1, 3.6], \Delta = .1$.944	.969
	$[3, 3.5], \Delta = .1$.922	.878
	$[3.5, 4], \Delta = .1$.922	.878
Class II	$[1.5, 4], \Delta = .5$.921	.876
	$[1, 3.5], \Delta = .5$.903	.817
	$(1.25, 2, 2.25, 2.75, 3.25, 3.75)$.899	.807
	$[2, 3.25], \Delta = .25$.687	.460
Class III	$[3, 4.25], \Delta = .25$.946	.981
	$[3, 5.5], \Delta = .5$.921	.876
	$[3.5, 6], \Delta = .5$.903	.817
	$(3.25, 4, 4.25, 4.75, 5.25, 5.75)$.811	.621
Class IV	$[0.5, 5.5], \Delta = 1$.891	.784
	$[1, 6], \Delta = 1$.597	.375
Class V	$[1, 2.25], \Delta = .25$.074	.016
	$[4.75, 6], \Delta = .25$.074	.016
	$[0.25, 1.50], \Delta = .25$.051	.000

*When visit times are evenly spaced, they are described by the notation $[a, b], \Delta$ where a and b denote the first and last cage visit and Δ denotes the length of time between cage visits.

Table 4: Power and asymptotic relative efficiency for arm-in-cage designs with $k = 6$ when $T_0 \sim TN(4.5, 0.5^2, [3, 6])$ and $T_1 \sim TN(1.5, 0.5^2, [0, 3])$. Power computed with $n = 0.064$ for each group.

Type of Design	Visit Times	Power	ARE
Class II	(.25, 1, 1.25, 1.75, 2.25, 2.75)	.950	—
	[1.75, 3], $\Delta = .25$.950	1.00
	[0.5, 3], $\Delta = .5$.399	.223
Class III	[3, 4.25], $\Delta = .25$.950	1.00
	(3.25, 4, 4.25, 4.75, 5.25, 5.75)	.950	1.00
	[3, 5.5], $\Delta = .5$.399	.223
	[3.5, 6], $\Delta = .25$.399	.223
Class IV	[0.5, 5.5], $\Delta = 1$.673	.447
	[2, 4.5], $\Delta = .5$.673	.447
	[2.5, 5], $\Delta = .5$.673	.447
	[1.5, 4], $\Delta = .5$.673	.447
	[1, 6], $\Delta = 1$.131	.052
Class V	[0.25, 1.50], $\Delta = .25$.057	.005
	[4.75, 6], $\Delta = .25$.053	.002

*When visit times are evenly spaced, they are described by the notation $[a, b], \Delta$ where a and b denote the first and last cage visit and Δ denotes the length of time between cage visits.

Table 5: Power and asymptotic relative efficiency for arm-in-cage designs with $k = 6$ when $T_0 \sim TN(4.5, .75^2, [3, 6])$ and $T_1 \sim TN(3.5, .75^2, [2, 5])$. Power computed with $n = 11.11$ for each group.

Type of Design	Visit Times	Power	ARE
Class I	$[3, 5], \Delta = .4$.950	—
	$[3.25, 4.5], \Delta = .25$.932	.932
	$[3.5, 4.75], \Delta = .25$.932	.932
	$[3, 4.25], \Delta = .25$.927	.899
Class II	$[2.5, 5], \Delta = .5$.948	.987
	$(2.25, 3, 3.25, 3.75, 4.25, 4.75)$.943	.965
	$[2, 4.5], \Delta = .5$.937	.932
Class III	$[3, 5.5], \Delta = .5$.948	.987
	$(3.25, 4, 4.25, 4.75, 5.25, 5.75)$.937	.936
	$[3.5, 6], \Delta = .5$.936	.932
Class IV	$[1, 6], \Delta = 1$.925	.890
	$[0.5, 5.5], \Delta = .5$.906	.827
Class V	$[4.75, 6], \Delta = .25$.617	.393
	$[2, 3.25], \Delta = .25$.617	.393
	$[5.2, 6], \Delta = .2$.307	.163
	$(0.5, 1.5, 2, 2.5, 5.5, 6)$.258	.132
	$[0.25, 1.50], \Delta = .25$.050	.000

*When visit times are evenly spaced, they are described by the notation $[a, b], \Delta$ where a and b denote the first and last cage visit and Δ denotes the length of time between cage visits.

Table 6: Power and asymptotic relative efficiency for arm-in-cage designs with $k = 6$ when $T_0 \sim U(3, 6)$ and $T_1 \sim U(2, 5)$. Power computed with $n = 12.99$ for each group.

Type of Design	Visit Times	Power	ARE
Class I	$[3, 5], \Delta = .4$.950	-
	$[3, 4.25], \Delta = .25$.803	.609
	$[3.25, 4.5], \Delta = .25$.707	.483
	$[3.5, 4.75], \Delta = .25$.707	.483
Class II	$[2.5, 5], \Delta = .5$.950	1.00
	$(2.25, 3, 3.25, 3.75, 4.25, 4.75)$.885	.769
	$[2, 4.5], \Delta = .5$.837	.667
Class III	$[3, 5.5], \Delta = .5$.950	1.00
	$[3.5, 6], \Delta = .5$.837	.667
	$(3.25, 4, 4.25, 4.75, 5.25, 5.75)$.837	.667
Class IV	$[1, 6], \Delta = 1$.950	1.00
	$[0.5, 5.5], \Delta = 1$.778	.571
Class V	$[4.75, 6], \Delta = .25$.732	.512
	$[2, 3.25], \Delta = .25$.732	.512
	$(0.5, 1.5, 2, 2.5, 5.5, 6)$.625	.400
	$[5.2, 6], \Delta = .2$.585	.367
	$[0.25, 1.50], \Delta = .25$.050	.000

*When visit times are evenly spaced, they are described by the notation $[a, b], \Delta$ where a and b denote the first and last cage visit and Δ denotes the length of time between cage visits.